(46) $\underset{x\to 0}{Lt} (\sin x)^{2\tan x}$.

Ans.→ $\underset{x\to 0}{Lt} (\sin x)^{2\tan x} \quad [0^0]$

Let $y = \underset{x\to 0}{Lt} (\sin x)^{2\tan x}$

Taking log both sides, we have

$$\log_e y = \underset{x\to 0}{Lt}\ 2\tan x \cdot \log \sin x$$

$$= \underset{x\to 0}{Lt}\ \frac{\log \sin x}{2\cot x} \quad \left[\frac{\infty}{\infty}\right]$$

Hence, from L' Hospital's Rule, we have

$$\log_e y = \underset{x\to 0}{Lt}\ \frac{\frac{1}{\sin x}\times \cos x}{-2\operatorname{cosec}^2 x}$$

$$= \underset{x\to 0}{Lt}\ \frac{\frac{\cos x}{\sin x}}{-2 \cdot 1/\sin^2 x}$$

$$\log_e y = \lim_{x\to 0} -2\sin x \cdot \cos x = 0$$

$$\log_e y = 0$$

$\therefore y = e^0 = 1$ Ans.

(47) Evaluate $\lim_{x\to\frac{\pi}{2}} (\tan x)^{\cos x}$.

Ans. $\to$ $\lim_{x\to\frac{\pi}{2}} (\tan x)^{\cos x} \; [\infty^0]$.

Let $y = \lim_{x\to\frac{\pi}{2}} (\tan x)^{\cos x}$

Taking log both sides, we have

$$\log_e y = \lim_{x\to\frac{\pi}{2}} \cos x \cdot \log(\tan x) = \lim_{x\to\frac{\pi}{2}} \frac{\log(\tan x)}{\frac{1}{\cos x}}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\log \tan x}{\sec x} \left[\frac{\infty}{\infty}\right].$$

Hence, from L' Hospital's Rule, we have,

$$\log_e y = \lim_{x\to\frac{\pi}{2}} \frac{\frac{1}{\tan x} \times \sec^2 x}{\sec x \cdot \tan x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\sec x}{\tan x} \times \frac{1}{\tan x} = \frac{\sec x}{\tan^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\frac{1}{\cos x}}{\frac{\sin^2 x}{\cos^2 x}}$$

$$\log_e y = \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\sin^2 x}$$

$$= \frac{0}{1} = 0$$

$$\log_e y = 0$$

$\therefore y = e^0 = 1$ Ans.

(48) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sec x)^{\cot x}$.

Ans.→ Let $y = \lim_{x \to \frac{\pi}{2}} (\sec x)^{\cot x}$ $[\infty^0]$

Taking log both sides, we have,

$$\log_e y = \lim_{x \to \frac{\pi}{2}} \cot x \cdot \log \sec x$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\log \sec x}{\tan x} \left[\frac{\infty}{\infty}\right]$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\frac{1}{\sec x} \cdot \cancel{\sec x} \cdot \tan x}{\sec^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\cos x \cdot \frac{\sin x}{\cancel{\cos x}}}{\cancel{\cos x}}$$

$$= \lim_{x \to \frac{\pi}{2}} \cos x \cdot \sin x$$

$$\log_e y = 1 \times 0 = 0$$

$\therefore \log_e y = 0$

$\therefore y = e^0 = 1$ Ans.

(49) Evaluate $\underset{x \to 0}{Lt} \left(\frac{1}{x}\right)^{\tan x}$.

Ans.→ Let $y = \underset{x \to 0}{Lt} \left(\frac{1}{x}\right)^{\tan x}$ $[\infty^0]$

Taking log both sides, we have;

$$\log_e y = \underset{x \to 0}{Lt} \tan x(-\log x) \quad [0 \times \infty]$$

$$= \underset{x \to 0}{Lt} \frac{-\log x}{\cot x} \quad \left[\frac{\infty}{\infty}\right]$$

Hence from L' Hospital's Rule, we have,

$$\log_e y = \underset{x \to 0}{Lt} \frac{-\frac{1}{x}}{-\operatorname{cosec}^2 x}$$

$$= \underset{x \to 0}{Lt} \frac{\cancel{-}\frac{1}{x}}{\cancel{-}\frac{1}{\sin^2 x}}$$

$$= \underset{x \to 0}{Lt} \frac{\sin^2 x}{x} \quad \left[\frac{0}{0}\right]$$

Hence, from L' Hospital's Rule, we have

$$\log_e y = \underset{x \to 0}{Lt} \frac{2 \sin x \cdot \cos x}{1}$$

$$= \frac{0 \times 1}{1} = 0$$

$\therefore \log_e^y = 0$

$\therefore y = e^0 = 1$ Ans.